MENIIT
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JEE MAINS-2019

10-01-2019 Online (Evening)

IMPORTANT INSTRUCTIONS

1. The question paper has three parts: **Physics**, **Chemistry and Mathematics** Each part has three sections.
2. Section 1 contains 8 questions. The answer to each question is a single digit integer ranging from 0 to 9 (both inclusive).
   Marking Scheme: +4 for correct answer and 0 in all other cases.
3. Section 2 contains 10 multiple choice questions with one or more than one correct option.
   Marking Scheme: +4 for correct answer, 0 if not attempted and –2 in all other cases.
4. Section 3 contains 2 “match the following” type questions and you will have to match entries in Column I with the entries in Column II.
   Marking Scheme: for each entry in Column I, +2 for correct answer, 0 if not attempted and – 1 in all other cases.

PART-A-PHYSICS

1. A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of $\vec{d} = 4\vec{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement?

(1) 9 J (2) 12 J (3*) 15 J (4) 10 J

Sol. $K.E. - 3 = \vec{F}.\vec{d}$

$K.E. = 3 + (3\hat{i} - 120\hat{j}) \times (4\hat{i})$

K.E. = 3 + 12 = 15 J

2. Consider the nuclear fission

$$Ne^{20} \longrightarrow 2He^{4} + C^{12}$$

Given that the binding energy/nucleon of Ne^{20}, He^{4} and C^{12} are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement:

(1) 8.3 MeV energy will be released

(2) energy of 12.4 MeV will be supplied

(3) energy of 3.6 MeV will be released

(4) energy of 11.9 MeV has to be supplied

Ans. No option is correct

Sol. $Q = (B.E.)_R - (B.E.)_P$

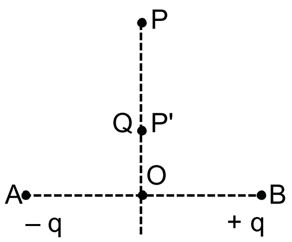
$= 20 \times 8.03 - (8 \times 7.07 + 12 \times 7.86)$

$= 160.6 - (56.56 + 94.32)$

$\therefore$ Q = +9.72 meV

9.72 MeV released.

3. Charges – q and + q located at A and B, respectively, constitute an electric dipole. Distance AB = 2a, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where OP = y and y >> 2a. The charge Q experiences an electrostatic force F. If Q is now moved along the equatorial line to P' such that OP' = $\left(\frac{y}{3}\right)$, the force on Q will be close to : $\left(\frac{y}{3} >> 2a\right)$



(1) $\frac{F}{3}$ (2*) 27 F (3) 9 F (4) 3 F

Sol. $\because$ $F \propto \frac{1}{r^3}$

Required force = 27 F.

4. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω is close to:

(density of water = 10^3 kg/m^3)

(1*) 1.25 rad s^{-1} (2) 2.50 rad s^{-1} (3) 3.75 rad s^{-1} (4) 5.00 rad s^{-1}

Sol. $A\rho gx = F_{restoring}$

$\pi r^2 \rho gx = n\omega^2 x$

$\therefore \quad \omega = \sqrt{\frac{\pi r^2 \rho g}{\rho v}}$

$= r\sqrt{\frac{\pi g}{v}} = 2.5 \times 10^{-2}\sqrt{\frac{3.14 \times 10}{310 \times 10^{-6}}}$

$= 2.5 \times 10^{-2} \times 10^2 \sqrt{10}$

$\omega = 2.5 \times \sqrt{10}$

$\therefore \quad f = \frac{25 \times \sqrt{10}}{2\pi} = 1.25$

5. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be (Assume that the highest frequency a person can hear is 20,000 Hz)

(1*) 6 (2) 7 (3) 5 (4) 4

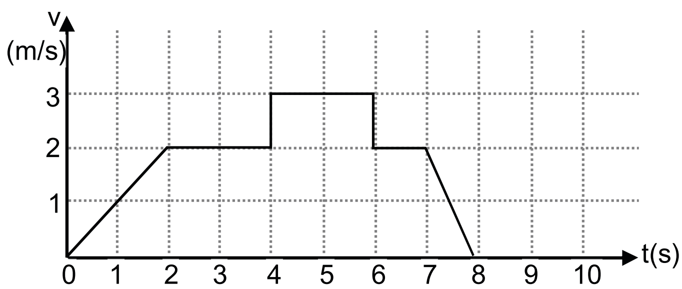
Sol. $\frac{250\text{kHz}}{1.5\text{kHz}} = 13.33$

$\therefore$ Possible hormones

1, 3, 5, 7, 9, 11, 13

i.e 6.

6. A particle starts from the origin at time t = 0 and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5s?



(1*) 9 m (2) 3 m (3) 10 m (4) 6 m

Sol. $r_t = 5 = \text{area}$

$$= \left(\frac{1}{2}\times 2\times 2 + 2\times 2 + 3\times 1\right)\text{m}$$

$= (2 + 4 + 3)$ m

$= 9$ m.

7. Two kg of a monoatomic gas is at a pressure of 4×10^4 N / m^2. The density of the gas is 8 kg / m^3. What is the order of energy of the gas due to its thermal motion?

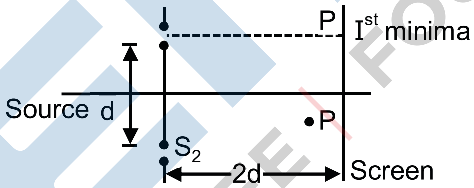
(1) 10^5 J (2*) 10^4 J (3) 10^3 J (4) 10^6 J

Sol. $E = \frac{1}{2} M V_m^2$

$$= \frac{1}{2}\times 2\times\left(\frac{3P}{\rho}\right)$$

$$= \frac{3\times 4\times 10^4}{8} = 1.5\times 10^4\,\text{J}$$

8. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1)?



(1) $\frac{\lambda}{2(5-\sqrt{2})}$ (2) $\frac{\lambda}{(5-\sqrt{2})}$ (3*) $\frac{\lambda}{2(\sqrt{5}-2)}$ (4) $\frac{\lambda}{(\sqrt{5}-2)}$

Sol. $\sqrt{(2d)^2 + (d)^2} - 2d = \frac{\lambda}{2}$

$\Rightarrow \quad \left(\sqrt{5}-2\right)d = \frac{\lambda}{2}$

$\Rightarrow \quad d = \frac{\lambda}{2\left(\sqrt{5}-2\right)}$

9. The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is:

(1) 540 J (2) 740 J (3*) 437.5 J (4) 637.5 J

Sol. $\frac{L\Delta T}{\Delta t} = 25$

$\Rightarrow \quad L = \frac{25\times 1}{15} = \frac{5}{3}$

$$\Delta U = \frac{1}{2}L\left(t_1^2 - t_2^2\right) = \frac{1}{2}\times\frac{5}{3}\times\left(25^2 - 10^2\right)$$

$$= \frac{5}{6}\times 525 = 437.5\ \text{J}$$

10. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation period of hoop and cylinder are T_h and T_c respectively, then

(1) $T_h = 1.5\ T_c$ (2*) $T_h = T_c$ (3) $T_h = 0.5T_c$ (4) $T_h = 2T_c$

Sol. $\because$ $T = 2\pi\sqrt{\frac{I}{\mu B}}$

$$\frac{T_h}{T_c} = \sqrt{\frac{I_R}{I_c}\times\frac{\mu_c}{\mu_h}}$$

$$= \sqrt{2\times\frac{1}{2}} = 1$$

$T_h = T_c$

11. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11 V is connected across it is:

(1) 11×10^{5} W (2) 11×10^{-4} W (3*) 11×10^{-5} W (4) 11×10^{-3} W

Sol. $R = \frac{R}{I^2} = \frac{4.4}{4\times10^{-6}}\Omega$

$$P' = \frac{V^2}{R} = \frac{11\times11\times4\times10^{-6}}{4.4}W$$

$= 11 \times 10^{-5}$ W

12. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license what broadcast frequency will you allot?

(1) 2250 kHz (2) 2900 kHz (3*) 2000 kHz (4) 2750 kHz

Sol. The interval between two carrier frequencies should be at least two times of AM frequency.

13. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

(1) 4.0 cm (2) 1 cm (3) 2 cm (4*) 3.1 cm

Sol. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \quad \frac{1.34}{v} - \frac{1}{-\infty} = \frac{0.34}{7.8}$

$\Rightarrow \quad v = \frac{1.34 \times 7.8}{0.34} nm = 3.074 \text{ cm}$

14. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is: (Take Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2})

(1) 2.4×10^4 m/s (2) 1.4×10^5 m/s (3*) 2.8×10^5 m/s (4) 3.8×10^4 m/s

Sol. $\frac{1}{2}mv^2 + \frac{2(-GMm)}{r} = 0$

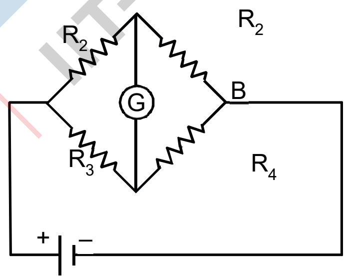
$V^2 = \frac{4GM}{r} = \frac{4 \times 6.67 \times 10^{-11} \times 3 \times 10^{31}}{2 \times 10^{11}}$

$V = 20\sqrt{2} \times 10^4 m/s$

$= 2.828 \times 10^5$ m/s

15. The Wheatstone bridge shown in figure here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80 Ω and 40Ω, respectively.

Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3, would be



(1) Red, Green, Brown (2) Brown, Blue, Black

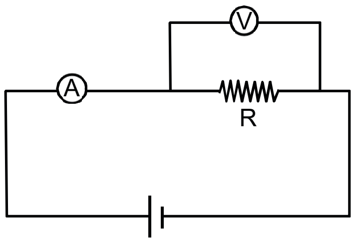
(3*) Brown, Blue, Brown (4) Grey, Black, Brown

Sol. $R_1 = 32 \times 10 = 320\ \Omega$

$R_3 = \frac{R_y}{R_2} \times R_1 = \frac{40 \times 320}{80} = 160\ \Omega$

$\therefore$ Colour code of R_3 be Brown, Blue, Brown.

16. The actual value of resistance R, shown in the figure is 30 Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is:



(1) 600 Ω (2*) 570 Ω (3) 35 Ω (4) 350 Ω

Sol. $\frac{30R}{R+30} = 30 \times 00.95$

$\Rightarrow$ R = 570 Ω

17. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is:

(1*) 508 pJ (2) 560 pJ (3) 692 pJ (4) 600 pJ

Sol. $W = \frac{Q^2}{2c} - \frac{Q^2}{2ck}$

$= \frac{Q^2}{2c}\left[1 - \frac{1}{k}\right]$

$= \frac{1}{2} \times 12 \times 100 \text{ pJ}\left(1 - \frac{1}{6.5}\right)$

$= \frac{12 \times 100 \times 11}{2 \times 13} \text{pJ} = 507.69 \text{ pJ}$

18. Two forces P and Q, of magnitude 2F and 3F, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is:

(1) 30° (2) 60° (3*) 120° (4) 90°

Sol. $2|\vec{P} + \vec{Q}| = |\vec{P} + 2\vec{Q}|$

$\Rightarrow$ 13 + 12 cos θ = 10 + 6 cos θ

$\cos = -\frac{1}{2}$

θ = 120°.

19. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures?

(1) 4264 ± 81 cm^3 (2*) 4260 ± 80 cm^3 (3) 4264.4 ± 81.0 cm^3 (4) 4300 ± 80 cm^3

Sol. $v = \pi R^2 h = \frac{\pi}{4} D^2 h$

$= 4260\ cm^2$

$\because \quad \frac{\Delta v}{v} = 2\frac{\Delta D}{D} + \frac{\Delta h}{h}$

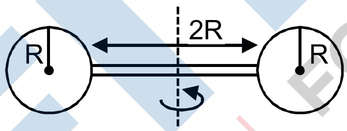
$= \left(2 \times \frac{0.1}{12.6} + \frac{0.1}{34.2}\right) v$

$= \frac{2 \times 426}{12.6} + \frac{426}{34.2}$

= 67.61 + 12.459 = 80.075

$\therefore \quad v = 4260 \pm 80\ cm^{30}$

20. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see fig.). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is:



(1) $\frac{152}{15} MR^2$ (2) $\frac{209}{15} MR^2$ (3*) $\frac{137}{15} MR^2$ (4) $\frac{17}{15} MR^2$

Sol. $I = 2\left[\frac{2}{5} MR^2 + M4R^2\right] + M\frac{4R^2}{12}$

$= MR^2\left[\frac{1}{3} + \frac{4}{5} + 8\right] = \frac{137}{15} MR^2$

21. A metal plate of area 1 × 10^{-4} m^2 is illuminated by a radiation of intensity 16 mW/m^2. The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photo electrons per second and their maximum energy, respectively, will be: [1 eV = 1.6 × 10^{-19} J]

(1*) 10^{11} and 5 eV (2) 10^{10} and 5 eV (3) 10^{12} and 5 eV (4) 10^{14} and 10 eV

Sol. $n = \frac{16 \times 10^{-3} \times 10^{-4}}{10 \times 10 \times 10^{-19}} = 1.6 \times 10^{-11}$

$(K.E.)_{max}$ = (10 – 5) eV = 5eV

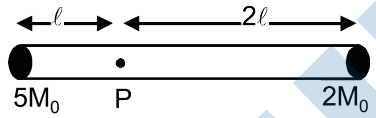
22. At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is:

(1*) 6.5×10^{-5} N (2) 3.6×10^{-5} N (3) 1.3×10^{-5} N (4) 1.8×10^{-5} N

Sol. $\tau = F \times 0.06 = 1.8 \times 0.012 \times 18 \times 10^{-6}$

$F = 6.48 \times 10^{-5}$

23. A rigid massless road of length 3ℓ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see fig.). When released from initial horizontal position, its instantaneous angular acceleration will be:



(1) $\frac{g}{2\ell}$ (2*) $\frac{g}{13\ell}$ (3) $\frac{g}{3\ell}$ (4) $\frac{7g}{3\ell}$

Sol. $\alpha = \frac{\tau}{\ell}$

$= \frac{5M_o g\ell - 4M_o g\ell}{5M_o\ell^2 + 2M_o 4\ell^2}$

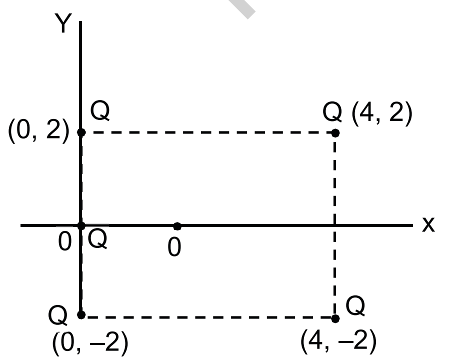
$= \frac{M_o g\ell}{13M_o\ell^2}$

$= \frac{g}{13\ell}$

24. Four equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), (4, – 2) and (0, – 2). The work required to put a fifth charge Q at the origin of the coordinate system will be:

(1) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$ (2*) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$ (3) $\frac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$ (4) $\frac{Q^2}{4\pi\varepsilon_0}$

Sol.



$$W = VQ = \frac{1}{4\pi\varepsilon_0}Q^2\left[\frac{1}{2}+\frac{1}{2}+\frac{2}{2\sqrt{5}}\right]$$

$$\therefore \quad = \frac{Q^2}{4\pi\varepsilon_0}\left[1+\frac{1}{\sqrt{5}}\right]$$

25. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5°C. (Specific heat of brass is 394 J kg^{-1} K^{-1})

(1*) 916 J kg^{-1} K^{-1} (2) 654 J kg^{-1} K^{-1} (3) 458 J kg^{-1} K^{-1} (4) 1232 J kg^{-1} K^{-1}

Sol. Heat loss = Heat gain

192 × S(100 – 21.5) = (128 × 0.394 + 240 × 4.2) (21.5 – 8.4)

192 × 78.5 × S = 1058.432 × 13.1

S = 0.91995 J/g K^{-1}

S = 919.95 J/kg K^{-1}

26. Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done by gas is close to: (Gas constant R = 8.31 J/mol. K)

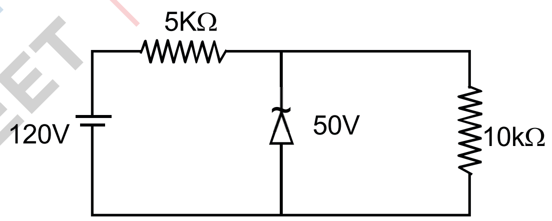
(1*) 291 J (2) 73 J (3) 581 J (4) 146 J

Sol. W = nRΔT

$$= \frac{1}{2}\times 8.31\times 70$$

= 290.85 J

27. For the circuit shown below, the current through the Zener diode is:



(1) Zero (2) 5 mA (3) 14 mA (4*) 9 mA

Sol. $i_{10k} = \frac{50}{10k} = 5\text{ mA}$

$i_{5k} = \frac{120-50}{5k} = 14\text{ mA}$

i_2 = (14 5)mA = 9 mA

28. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is:

(1) $\frac{3}{8}\pi$ (2) $\frac{4\pi}{3}$ (3*) $\frac{8\pi}{3}$ (4) $\frac{7}{3}\pi$

Sol. $|v_4| = |a_4|$

$\Rightarrow \left(w\sqrt{A^2 - x^2}\right)_4 = \left(w^2 x\right)_4$

$\Rightarrow w\sqrt{25-16} = w^2 \times 4$

$\Rightarrow w = \frac{3}{4}$

$T = \frac{2\pi}{w} = 2\pi\frac{4}{3} = \frac{8\pi}{3}$

29. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $\left(\vec{A}+\vec{B}\right)$ is 'n' times the magnitude of $\left(\vec{A}-\vec{B}\right)$. The angle between $\vec{A}$ and $\vec{B}$ is:

(1) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$ (2*) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (3) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (4) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$

Sol. $\left|\vec{A}+\vec{B}\right| = n\left|\vec{A}-\vec{B}\right|$

$\Rightarrow A^2 + B^2 + 2AB\cos\theta$

$= n^2(A^2 + B^2 - 2AB\cos\theta)$

$\Rightarrow \cos\theta(1+n^2) = \frac{2a^2(n^2-1)}{2a^2}$ $[A = B = a$

$\cos\theta = \frac{n^2-1}{n^2+1}$

30. The electric field of a plane polarized electromagnetic wave in free space at time t = 0 is given by an expression $\vec{E}(x,y) = 10\hat{j}$ cos [(6x + 8z)]. The magnetic field $\vec{B}$ (x, z, t) is given by:

(c is the velocity of light)

(1) $\frac{1}{c}\left(6\hat{k}+8\hat{i}\right)\cos[(6x+8z-10ct)]$ (2) $\frac{1}{c}\left(6\hat{k}+8\hat{i}\right)\cos[(6x-8z+10ct)]$

(3*) $\frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\cos[(6x+8z-10ct)]$ (4) $\frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\cos[(6x+8z+10ct)]$

Sol. $\vec{E} = 10\hat{j}\cos(6x+8z-10ct)$

$B_o = \frac{E_o}{C} = \frac{10}{C}$

W = 10 C

$\because \quad \hat{E}\times\hat{B}=\hat{C}$

$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ 0 & 1 & 0 \\ B_x & B_y & B_z \end{vmatrix} = \frac{6\hat{i}+8\hat{j}}{10}$$

$\Rightarrow \quad B_z\hat{i}-0\hat{j}-B_x\hat{k}=\frac{3}{5}\hat{i}+\frac{4}{5}\hat{j}$

$B_z=\frac{3}{5}, B_y=0, B_x=\hat{j}$

$\therefore \quad \bar{B}=\frac{1}{C}\left(-8\hat{i}+6\hat{k}\right)\cos\left(6x+8z+10ct\right)$

PART-B-CHEMISTRY

31. 5.1g NH_4SH is introduced in 3.0 L evacuated flask at 327°C. 30% of the solid NH_4SH decomposed to NH_3 and H_2S as gases. The K_P of the reaction at 327°C is (R = 0.082 L atm mol^{-1} K^{-1}, Molar mass of S = 32 g mol^{-1}, Molar mass of N = 14 g mol^{-1})

(1) 1×10^{-4} atm^2 (2) 4.9×10^{-3} atm^2 (3*) 0.242 atm^2 (4) 0.242×10^{-4} atm^2

Sol. $NH_4HS\ (s) \rightleftharpoons NH_3\ (s) + H_2S\ (s)$

t=0 0. 1 mole

t_{eq} 0.1–x x x

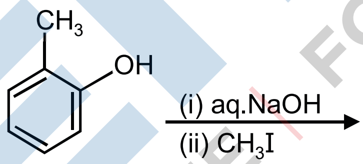
$$\because x = \frac{30}{100} \times 0.1 mol = 3 \times 10^{-2} mol$$

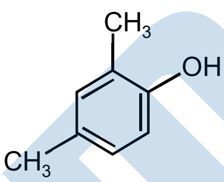
$$P_{NH_3} = P_{H_2S} = \frac{3 \times 10^{-2} \times 0.082 \times 600}{3} atm$$

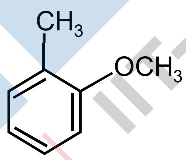
= 0.492 atm

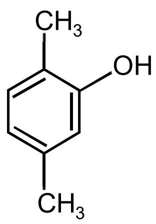
$$K_P = \frac{(P_{NH_3})^1 (P_{H_2S})^1}{1} = 0.242\ atm^2$$

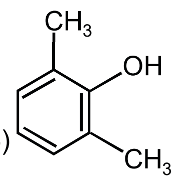
32. The major product of the following reaction. :

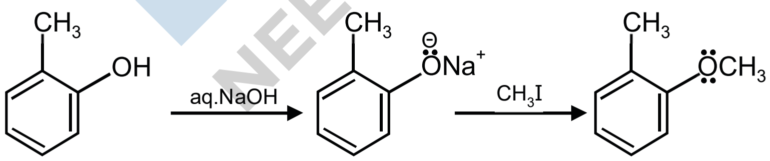


(1) 

(2*) 

(3) 

(4) 

Sol. 

33. The ground state energy of hydrogen atom is – 13.6 eV. The energy of second excited state of He^+ ion in eV is -

(1) – 3.4 (2*) – 6.04 (3) – 54.4 (4) – 27.2

Sol. Second excited state $\Rightarrow$ n = 3

$$E = -13.6 \times \frac{2^2}{3^2} = -6.04\ eV$$

34. The 71[st] electron of an element X with an atomic number of 71 enters into the orbital:

(1) 6s (2) 6p (3) 5d (4*) 4f

Sol. The electron configuration is $[Xe]4f^{14}5d^{1}6s^{2}$

35. A compound of formula A_2B_3 has the hcp lattice. Which atom forms the hcp lattice and what fraction of tetrahedral voids is occupied by the other atoms:

(1) hcp lattice - A, $\frac{2}{3}$ Tetrahedral voids-B

(2) hcp lattice - B, $\frac{2}{3}$ Tetrahedral voids-A

(3) hcp lattice - A, $\frac{1}{3}$ Tetrahedral voids-B

(4*) hcp lattice - B, $\frac{1}{3}$ Tetrahedral voids-A

Sol. Should be checked by options,

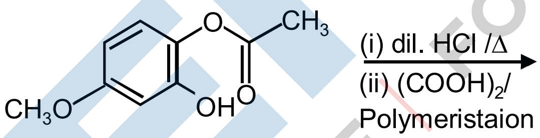
when B in HCP $\Rightarrow (Z)_B = 6$

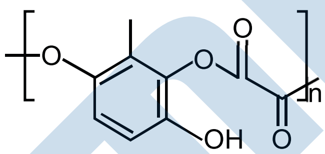
Total T.V. = 12

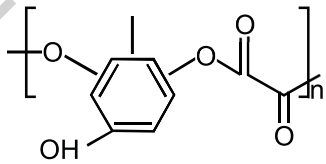
If A in $\frac{1}{3}$rd of T.V. $\Rightarrow (Z)_A = \frac{1}{3} \times 12 = 4$

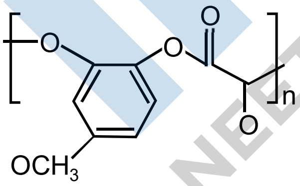
so formula, A_4B_6 or A_2B_3

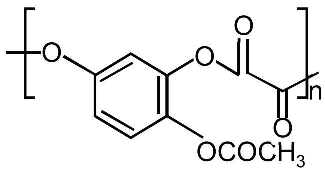
36. The major product of the following reaction is:

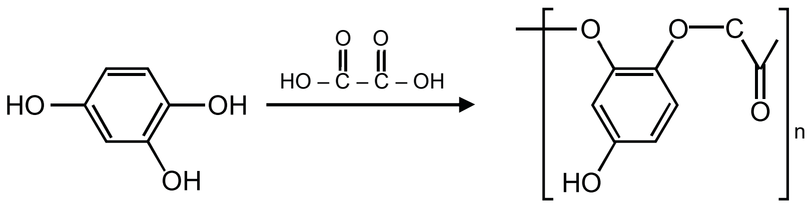


(1)


(2*)


(3)


(4)


Sol.


37. Which is the most suitable reagent for the following transformation?

$$CH_3\text{-}CH{=}CH\text{-}CH_2\text{-}\underset{}{\overset{OH}{\overset{|}{C}}}H\text{-}CH_3 \rightarrow CH_3\text{–}CH{=}CH\text{–}CH_2CO_2H$$

(1) Tollen's reagent (2) CrO_2Cl_2/CS_2 (3*) I_2/NaOH (4) alkaline $KMnO_4$

Sol. Iodoform reaction can be used for this transformation.

38. The electrolytes usually used in the electroplating of gold and silver, respectively, are:

(1) $[Au(CN)_2]^-$ and $[AgCl_2]^-$ (2) $[Au(NH_3)_2]^+$and $[Ag(CN)_2]^-$

(3) $[Au(OH)_4]^-$and $[Ag(OH)_2]^-$ (4*) $[Au(CN)_2]^-$and $[Ag(CN)_2]^-$

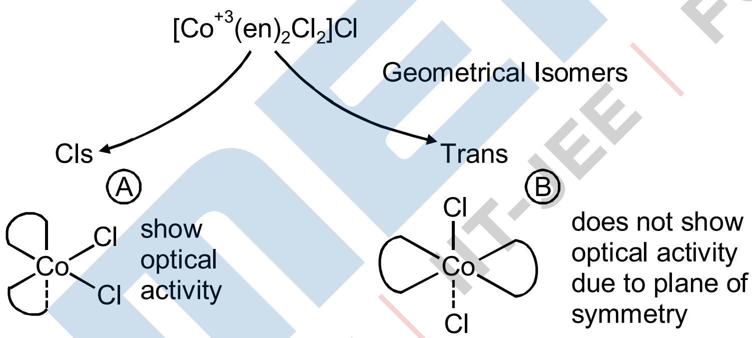
Sol. $[Au(CN)_2]^-$ and $[Ag(CN)_2]^-$ both are soluble complexes.

39. A reaction of cobalt (III) chloride and ethylenediamine in a 1: 2 mole ratio generates two isomeric products A (violet coloured) and B(green coloured). A can show optical activity, but, B is optically inactive. What type of isomers does A and B represent?

(1) Linkage isomers (2*) Geometrical isomers

(3) Ionisation isomers (4) Coordination isomers

Sol. Cobalt (III) chloride on reaction with ethylenediamine in ratio 1 : 2 2 isomeric products complexes A and B



40. For an elementary chemical reaction, $A_2 \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2A$, the expression for $\frac{d[A]}{dt}$ is:

(1) $k_1[A_2] + k_{-1}[A]^2$ (2*) $2k_1[A_2] - 2k_{-1}[A]^2$

(3) $2k_1[A_2] - k_{-1}[A]^2$ (4) $k_1[A_2] - k_{-1}[A]^2$

Sol. Rate of forward reaction, $R_F = k_1.[A_2]^1 = \frac{1}{2}\left(\frac{d[A]}{dt}\right)$

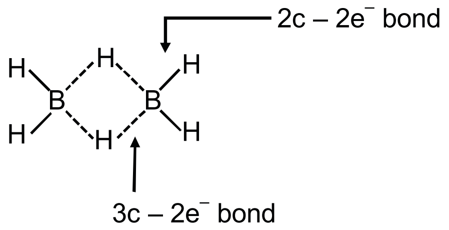
Rate of backward reaction, $R_b = k_{-1}[A]^2 = \frac{1}{2}\left(-\frac{d[A]}{dt}\right)$

so $\left(\frac{d[A]}{dt}\right)_{net} = 2k_1[A_2]^1 - 2k_{-1}[A]^2$

41. The number of 2-centre-2-electron and 3-centre-2-electron bonds in B_2H_6, respectively, are:

(1) 2 and 2 (2*) 4 and 2 (3) 2 and 4 (4) 2 and 1

Sol.



42. Among the following reactions of hydrogen with halogens, the one that requires a catalyst is:

(1*) $H_2 + I_2 \rightarrow 2HI$ (2) $H_2 + Br_2 \rightarrow 2HBr$ (3) $H_2 + Cl_2 \rightarrow 2HCl$ (4) $H_2 + F_2 \rightarrow 2HF$

Sol. First reaction will be requiring a catalyst among halogens oxidizing power decrease down the group.

43. Elevation in the boiling point for 1 molal solution of glucose is 2K. The depression in the freezing point for 2 molal solution of glucose in the same solvent is 2K. The relation between K_b and K_f is:

(1) $K_b = K_f$ (2) $K_b = 0.5K_f$ (3*) $K_b = 2K_f$ (4) $K_b = 1.5K_f$

Sol. $\Delta T_b = K_b.m$; $\Delta T_f = K_f.m$

$2 = K_b.1$ $2 = K_f.2$

$\Rightarrow K_b = 2.K_f$

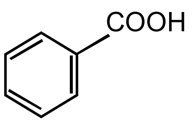
44. Sodium metal on dissolution in liquid ammonia gives a deep blue solution due to the formation of:

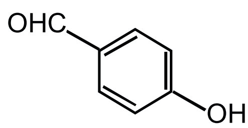
(1) sodamide (2) sodium-ammonia complex

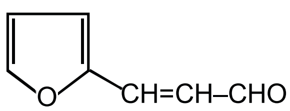
(3*) ammoniated electrons (4) sodium ion-ammonia complex

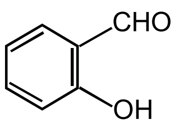
Sol. $Na(s) + (x+y)NH_3 \longrightarrow Na^+(NH_3)_x + \underset{\text{ammonlated electrons}}{\underset{\text{Bluecolour}}{e^-(NH_3)_y}}$

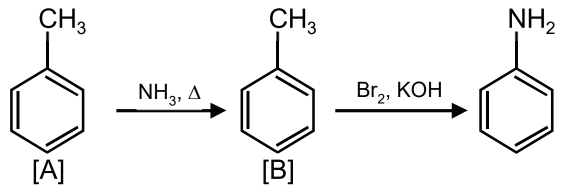
45. An aromatic compound 'A' having molecular formula $C_7H_6O_2$ on treating with aqueous ammonia and heating forms compound 'B'. The compound 'B' on reaction with molecular bromine and potassium hydroxide provides compound 'C' having molecular formula C_6H_7N. The structure of 'A' is:

(1*) 

(2) 

(3) 

(4) 

Sol. 

46. The reaction that is NOT involved in the ozone layer depletion mechanism in t he stratosphere is :

(1) $HOCl(g) \xrightarrow{h\nu} \dot{O}H(g) + \dot{C}l(g)$

(2) $CF_2Cl_2(g) \xrightarrow{uv} \dot{C}l(g) + \dot{C}F_2Cl(g)$

(3) $Cl\dot{O}(g) \longrightarrow \dot{C}l(g) + O_2(g)$

(4*) $CH_4 + 2O_3 \longrightarrow 3CH_2 = O + 3H_2O$

Sol. CH_4 is not present in stratosphere.

47. In the cell Pt(s) | H_2(g,1bar) | HCl(aq)|AgCl(s)| Ag(s) |Pt(s) the cell potential is 0.92 V when a 10^{-6} molal HCl solution is used. The standard electrode potential of ($AgCl/Ag,Cl^-$) electrode is :

[Given: $\frac{2.303RT}{F}$ = 0.06 V at 298 K]

(1) 0.94 V (2*) 0.20 V (3) 0.76 V (4) 0.40 V

Sol. Anode: $\frac{1}{2}H_2(s) \rightarrow H^+(a) + e^-$; E° = 0.0 V

Cathode: $AgCl(s) + e^- \rightarrow Ag(s) + Cl^-(a)$; E° = x V

Net cell reaction,

$$\frac{1}{2}H_2(s) + AgCl(s) \rightarrow H^+(a) + Ag(s) + Cl^-(a)$$

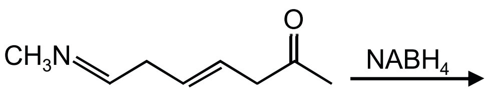
$$E_{cell} = E^{o}_{cell} - \frac{0.06}{n}\log\frac{[H^+][Cl^-]}{(P_{H_2})^{1/2}}$$

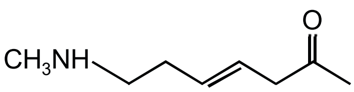
$$0.92 = (0 + x) - \frac{0.06}{1}\log\frac{(10^{-6})(10^{-6})}{(1)^{1/2}}$$

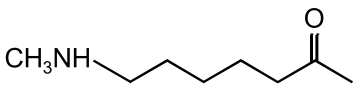
$$0.92 = x - 0.06 \times (-12)$$

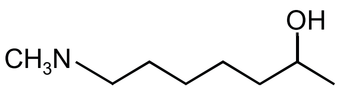
$$x = 0.2 \text{ V}$$

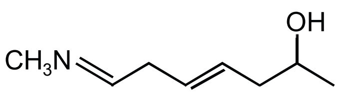
48. The major product of the following reaction:



(1) 

(2) 

(3*) 

(4) 

Sol. $NaBH_4$ reduces both carbonyl group and imine.

49. Which of the following tests cannot be used for identifying amino acids?

(1*) Barfored test (2) Biuret test (3) Ninhydrin test (4) Xanthoproteic test

Sol. Barfoed test is used to detect monosaccirides.

50. The difference in the number of unpaired electrons of a metal ion in its high-spin and low-spin octahedral complexes is two. the metal ion is:

(1*) Co^{2+} (2) Fe^{2+} (3) Ni^{2+} (4) Mn^{2+}

Sol. Co^{2+} high spin $t_{2g}^5 e_g^2$ '3' unpaired electrons

Co^{2+} low spin $t_{2g}^6 e_g^1$ '1' unpaired electron

Difference is 3 – 1 = 2

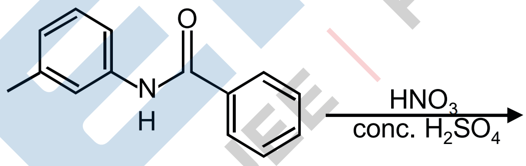
51. In the reaction of oxalate with permanganate in acidic medium, the number of electrons involved in producing one molecule of CO_2 is:

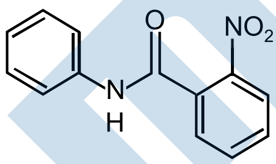
(1) 5 (2) 2 (3) 10 (4*) 1

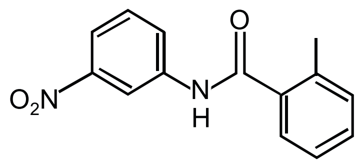
Sol. $C_2O_4^{2-} \longrightarrow 2CO_2 + 2e^-$

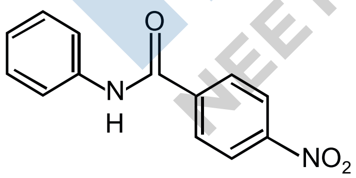
so for 1 molecule of CO_2 = 1 electron

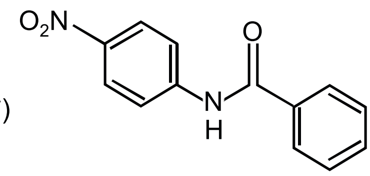
52. What will be the major product in the following mononitration reaction?



(1) 

(2) 

(3) 

(4*) 

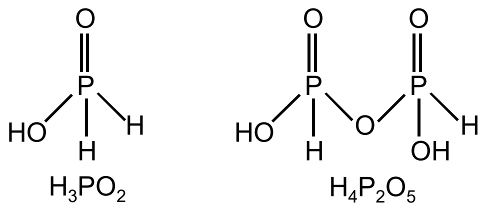
Sol. 'EAS' first ring is activated and second is deactivated NO_2^+ attack at para position of activated ring.

53. The pair that contains two P – H bonds in each of the oxoacids is:

(1*) H_3PO_2 and $H_4P_2O_5$ (2) H_3PO_3 and H_3PO_2

(3) $H_4P_2O_5$ and H_3PO_3 (4) $H_4P_2O_5$ and $H_4P_2O_6$

Sol.



54. The correct match between item I and item II is:

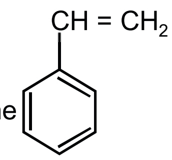
	item I (compound)		item II (reagent)
(A)	Lysine	(P)	1-naphthol
(B)	Furfural	(Q)	ninhydrin
(C)	Benzyl alcohol	(R)	$KMnO_4$
(D)	Styrene	(S)	Ceric ammonium nitrate

(1*) (A) → (Q); (B) → (P); (C) → (S); (D) → (R)
(2) (A) → (R); (B) → (P); (C) → (Q); (D) → (S)
(3) (A) → (Q); (B) → (R); (C) → (S); (D) → (P)
(4) (A) → (Q); (B) → (P); (C) → (R); (D) → (S)

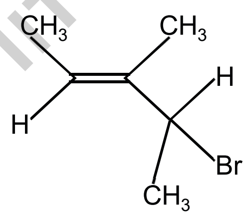
Sol. Lysine is an amino acid nindydrin test is used for amino acids.

Furfural reacts with 1-napthol to give violet colouration.

Benzyl alcohol undergoes reaction with ceric ammonium nitrate to give red colouration.

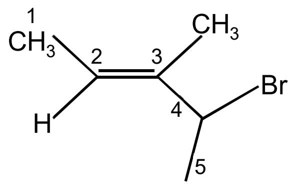
Styrene  discharges color of $KMnO_4$

55. What is the IUPAC name of the following compound?



(1) 2-Bromo-3-methylpent-3-ene
(2) 3-Bromo-3-methyl-1, 2-dimethylprop-1-ene
(3*) 4-Bromo-3-methylpent-2-ene
(4) 3-Bromo-1,2-dimethylbut-1-ene

Sol.



4-Bromo-3-methyl pentane

56. An ideal gas undergoes isothermal compression from 5 m^3 to 1 m^3 against a constant external pressure of 4 Nm^{-2}. Heat released in this process is used to increase the temperature of 1 mole of Al. If molar heat capacity of Al is 24 J mol^{-1} K^{-1}, the temperature of Al increases by:

(1) 2K (2) 3/2 K (3*) 2/3 K (4) 1K

Sol. Isothermal $\Rightarrow \Delta U = 0$

so $q = -w_{irr}$

$= +P_{ext.}(V_2 - V_1)$

$= 4(1-5)$

$= -16$ J

For Al

$q = n.C_m.\Delta T$

$16 = 1 \times 24 \times \Delta T$

$\Delta T = \frac{2}{3} K$

57. The amount of sugar ($C_{12}H_{22}O_{11}$) required to prepare 2L of its 0.1 M aqueous solution is:

(1) 17.1 g (2) 34.2 g (3*) 68.4 g (4) 136.8 g

Sol. n = 0.1 × 2 = 0.2 mole

Mass of sugar = 0.2 × 342

= 68.4 g

58. Haemoglobin and gold sol are examples of:

(1) negatively charged sols

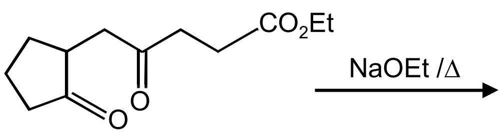
(2) positively charged sols

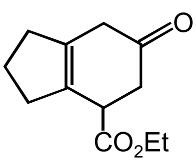
(3) negatively and positively charged sols, respectively

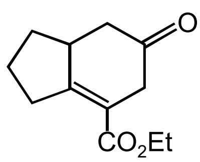
(4*) positively and negatively charged sols, respectively

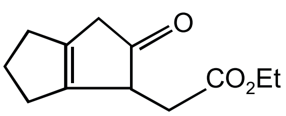
Sol. Theory based

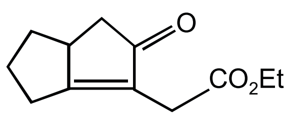
59. The major product obtained in the following reaction is:



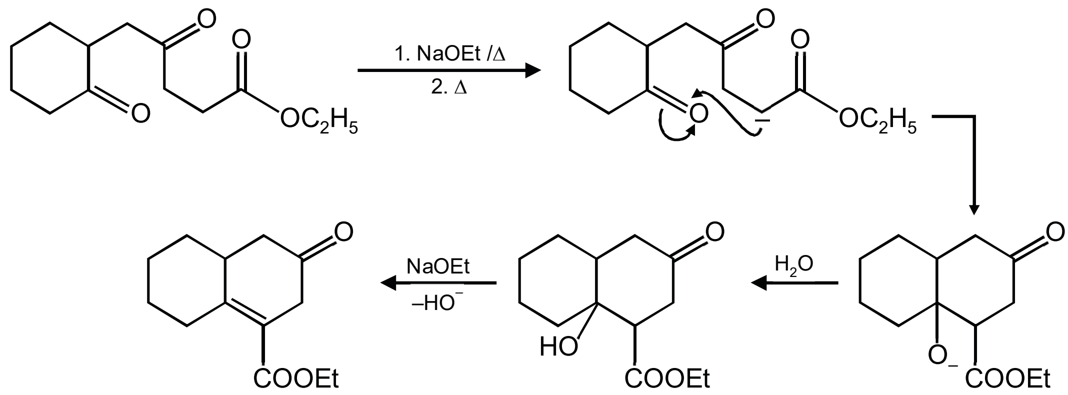
(1) 

(2*) 

(3) 

(4) 

Sol.



60. The process with negative entropy change is:

(1) Sublimation of dry ice

(2) Dissolution of iodine in water

(3) Dissociation of $CaSO_4$(s) to CaO(s) and SO_3(g)

(4*) Synthesis of ammonia from N_2 and H_2

Sol. $N_2\ (s) + 3H_2(s) \rightarrow 2NH_3\ (s)$

$\because \Delta n_g < 0 \Rightarrow \Delta S < 0$

PART-C-MATHEMATICS

61. The curve amongst the family of curves represented by the differential equation, $(x^2 - y^2)\,dx + 2xy\,dy = 0$ which passes through (1, 1) is

(1) a hyperbola with transverse axis along the x-axis.

(2*) a circle with centre on the x-axis.

(3) a circle with centre on the y-axis.

(4) an ellipse with major axis along the y-axis

Sol. $x^2dx + 2xydy + y^2dx = 0$

$x^2dx = y^2dx - 2xy\,dy$

$(x^2 - y^2)\,dx + 2xy\,dy = 0$

$$dx = -\left(\frac{x.2y\,dy - y^2dx}{x^2}\right)$$

Integrals

$$x = -\frac{y^2}{x} + c$$

$x^2 + y^2 = cx$

62. A helicopter is flying along the curve given by $y - x^{\frac{3}{2}} = 7$, $(x \geq 0)$. A solider positioned at the point 7 $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is

(1) $\frac{1}{2}$ (2) $\frac{\sqrt{5}}{6}$ (3) $\frac{1}{3}\sqrt{\frac{7}{3}}$ (4*) $\frac{1}{6}\sqrt{\frac{7}{3}}$

Sol. $y = x^{3/2} - 2$ $\frac{dy}{dx} = \frac{3}{2}\sqrt{x}$

Slope of normal $= -\frac{2}{3\sqrt{x}}$

Let point is $\left(x_1, x_1^{3/2} - 2\right)$

$\therefore$ Normal $y - \left(x_1^{3/2} - 2\right) = \frac{-2}{3\sqrt{x_1}}(x - x_1)$

Now put (1, 7) and solve it.

$$\Rightarrow x_1 = \frac{1}{3}$$

$$\therefore P \Rightarrow \left(\frac{1}{3}, 7 + \frac{1}{3\sqrt{3}}\right), A \Rightarrow (1,7)$$

$$\therefore AD = \frac{1}{6}\sqrt{\frac{7}{3}}$$

63. Let $a_1, a_2, a_3, \ldots\ldots, a_{10}$ be in G.P. with $a_i > 0$ for i = 1, 2, ……., 10 and S be the set of pairs (r, k), $r, k \in N$ (the set of natural numbers) for which $\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$. Then the number of elements in S, is

(1) 4 (2) 2 (3*) infinitely many (4) 10

Sol. For any value of r determinant is zero.

64. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{[x]+[\sin x]+4}$, where [t] denotes the greatest integer less than or equal to t, is

(1) $\frac{1}{12}(7\pi + 5)$ (2) $\frac{1}{12}(7\pi - 5)$ (3*) $\frac{3}{20}(4\pi - 3)$ (4) $\frac{3}{10}(4\pi - 3)$

Sol. $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x]+[\sin x]+4}$

$$= \int_{-\pi/2}^{0} \frac{dx}{[x]+-1+4} + \int_{0}^{\frac{\pi}{2}} \frac{dx}{[x]+4}$$

$$= \int_{-\pi/2}^{-1} \frac{dx}{-2-1+4} + \int_{-1}^{0} \frac{dx}{-1-1+4} + \int_{0}^{1} \frac{dx}{4} + \int_{1}^{\pi/2} \frac{dx}{1+4}$$

$$= -1 + \frac{\pi}{2} + 2 + \frac{1}{4} + \frac{1}{5}\left(\frac{\pi}{2} - 1\right)$$

$$= 3\frac{\pi}{5} - \frac{9}{20}$$

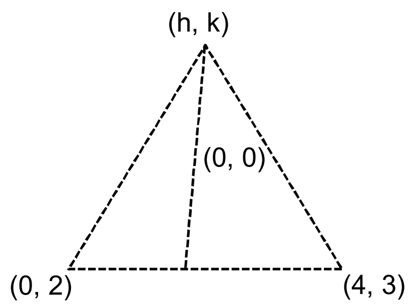
65. Two vertices of a triangle are (0, 2) and (4, 3). If its orthocentre is at the origin, then its third vertex lies in which quadrant?

(1) third (2) fourth (3*) second (4) first

Sol. $\frac{k-3}{h-4} = 0$ $k = 3$

$\frac{k}{h} = \frac{4-0}{3-2}$ $-4h = k$

$h = \frac{-3}{4}$



66. If $\sum_{r=0}^{25} \left\{ {}^{50}C_r \cdot {}^{50-r}C_{25-r} \right\} = K\,({}^{50}C_{25})$, then K is equal to

(1) 2^{25} (2) $(25)^2$ (3) 2^{24} (4*) $2^{25} - 1$

Sol. $\sum_{r=1}^{25} \frac{\lfloor 50}{\lfloor r \lfloor 50-r} \times \frac{\lfloor 50-r}{\lfloor 25-r \lfloor 25}$

$= \sum_{r=1}^{25} \frac{\lfloor 50}{\lfloor r \lfloor 25-r \lfloor 25}$

$= \frac{\lfloor 50}{\lfloor 25} \sum_{r=1}^{25} \frac{1}{\lfloor r \lfloor 25-r}$

$= \frac{\lfloor 50}{\lfloor 25 \lfloor 25} \sum_{r=1}^{25} {}^{25}C_r = {}^{50}C_{25}\left(2^{25}-1\right)$

67. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$x + 3y + 7z = 0$

$-x + 4y + 7z = 0$

$(\sin 3\theta)\, x + (\cos 2\theta)\, y + 2z = 0$

has a non-trivial solution is

(1) four (2*) two (3) one (4) three

Sol. $\begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$

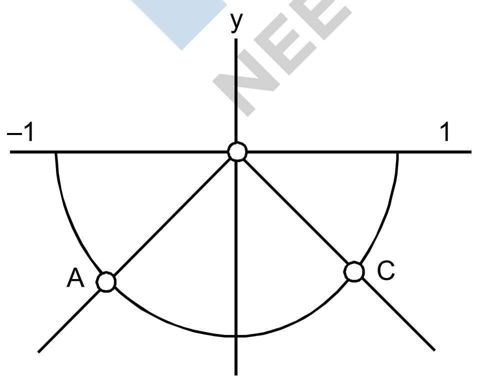
$7 \sin 3\theta + 14 \cos 2\theta - 14 = 0$

$\sin 3\theta + 2\cos 2\theta - 2 = 0,\ \sin\theta = \frac{1}{2}$

68. Let $f : (-1, 1) \to R$ be a function defined by $f(x) = \max.\left\{-|x|, -\sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly

(1) five elements (2) one element (3*) three elements (4) two elements

Sol. A, B, C are sharp edges



69. The tangent to the curve, $y = xe^{x^2}$ passing through the point (1, e) also passes through the point

(1) $\left(\frac{5}{3}, 2e\right)$ (2*) $\left(\frac{4}{3}, 2e\right)$ (3) (3, 6e) (4) (2, 3e)

Sol. $y = xe^{x^2}$

(1, e) lies on this

Now $\frac{dy}{dx} = xe^{x^2}.2x + e^{x^2}.1$

Put x = 1

m = 2e + e = 3e

Equation of tangent at (1, e)

y – e = 3e(x – 1)

y – e = 3ex – 3e

y = 3ex – 2e

$\left(\frac{4}{3}, 2e\right)$ satisfies it

∴ Answer is B

70. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is

(1) $\frac{22}{23}$ (2) $\frac{23}{22}$ (3) $\frac{21}{19}$ (4) $\frac{19}{21}$

Sol. $\cot\left[\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2P\right)\right]$

$= \cot\left[\sum_{n=1}^{19}\cot^{-1}\left(1+n^2+n\right)\right]$

$= \cot\left[\sum_{n=1}^{19}\tan^{-1}\left(\frac{1}{1+n^2+n}\right)\right]$

$= \cot\left[\sum_{n=1}^{19}\tan^{-1}(n+1) - \tan^{-1}1\right]$

= cot [$\tan^{-1}$ 20 – $\tan^{-1}$1]

$= \cot\left(\tan^{-1}\frac{19}{21}\right)$

$\Rightarrow \frac{21}{19}$

71. If $\int_0^x f(t)\,dt = x^2 + \int_x^1 t^2 f(t)\,dt$, then $f'\left(\frac{1}{2}\right)$ is

(1) $\frac{4}{5}$ (2) $\frac{18}{25}$ (3) $\frac{6}{25}$ (4*) $\frac{24}{25}$

Sol. Differentiability we get f (x) = 2x – x^2 f (x)

$$f(x) = \frac{2x}{1+x^2} \Rightarrow f''(x) = 2\frac{(1-x^2)}{(1+x^2)^2}$$

$$f'\left(\frac{1}{2}\right) = \frac{24}{25}$$

72. With the usual notation, in ΔABC, if ∠A + ∠B = 120°, a = $\sqrt{3}$ + 1 and b = $\sqrt{3}$ – 1, then the ratio ∠A : ∠B, is

(1) 9 : 7 (2*) 7 : 1 (3) 3 : 1 (4) 5 : 3

Sol. $a = \sqrt{3} + 1$

$b = \sqrt{3} - 1$

$$\frac{\sin A}{\sin B} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{3+1+2\sqrt{3}}{2} = 2+\sqrt{3}$$

$$\frac{\sin A}{\sin(120-A)} = \sqrt{3}+2$$

$$\frac{\sin A}{\sin 12 \cos A - \cos 12 \sin A} = \sqrt{3}+2$$

$$\frac{1}{\frac{\sqrt{3}}{2}\cot A + \frac{1}{2}} = \sqrt{3}+2$$

$$\frac{\sqrt{3}\cot A + 1}{2} = \frac{1}{\sqrt{3}+2}$$

$$= \frac{\sqrt{3}-2}{-1}$$

$$\frac{\sqrt{3}\cot A + 1}{2} = -\sqrt{3}+2$$

$$\sqrt{3}\cot A = 4 - 2\sqrt{3} - 1$$

$$\sqrt{3}\cot A = 3 - 2\sqrt{3}$$

$$\cot A = \sqrt{3} - 2$$

$$-\cot A = 2 - \sqrt{3} = \tan 15$$

∴ A = 105°

∴ B = 15°

73. Let N be the set of natural numbers and two functions f and g be defined as f, g: N → N such that

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if n is odd} \\ \frac{n}{2}, & \text{if n is even} \end{cases}$$ and $g(n) = n - (-1)^n$. Then fog is

(1) both one-one and onto
(2) neither one-one nor onto
(3*) onto but not one-one
(4) one-one but not onto

Sol. $\left.\begin{array}{l} f(g(1)) = 1 \\ f(g(2)) = 1 \end{array}\right\}$ Many one

$f(g(2k)) = k$

$f(g(2k+1)) = k+1$

∴ Onto

74. If the probability of hitting a target by a shooter, in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target atleast once is greater than $\frac{5}{6}$, is

(1) 3
(2) 4
(3*) 5
(4) 6

Sol. $1 - \left(\frac{2}{3}\right)^n > \frac{5}{6}$

$\left(\frac{2}{3}\right)^n < \frac{1}{6}$

⇒ n = 5

75. The value of $\cos\left(\frac{\pi}{2^2}\right)\cdot\cos\left(\frac{\pi}{2^3}\right)\cdot\ldots\ldots\cdot\cos\left(\frac{\pi}{2^{10}}\right)\cdot\sin\left(\frac{\pi}{2^{10}}\right)$ is

(1) $\frac{1}{2}$
(2*) $\frac{1}{512}$
(3) $\frac{1}{256}$
(4) $\frac{1}{1024}$

Sol. Using formula $\frac{\sin 2^n A}{2^n \sin A}$ = cosA cos2A $\cos 2^2$ A..........$\cos 2^{n1}$ A

76. If the area of an equilateral triangle inscribed in the circle $x^2 + y^2 + 10x + 12y + c = 0$ is 27 $\sqrt{3}$ s units, then c is equal to:

(1) – 25
(2) 20
(3) 13
(4*) 25

Sol. $r = \sqrt{25+36-c} = \sqrt{36}$

c = 25

77. Let $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is

(1) – 3 (2*) – 4 (3) 4 (4) 3

Sol. $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$

$\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$

$\vec{\alpha}$ and $\vec{\beta}$ are collinear

$$\begin{vmatrix} \lambda - 2 & 1 \\ 4\lambda - 2 & 3 \end{vmatrix} = 0$$

$3\lambda - 6 - 4\lambda + 2 = 0$

$-\lambda - 4 = 0$

$\lambda = -4$

78. On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, $x + y + z = 2$?

(1) $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$ (2) $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$

(3*) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$ (4) $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

Sol. Put $(2\lambda + 4, 2\lambda + 5, \lambda + 3)$ in $x + y + z = 2$

$2\lambda + 4 + 2\lambda + 5 + \lambda + 3 = 2$

$5\lambda = -10$ $\lambda = -2$

P(0, 1, 1)

Now put in options

Answer is C

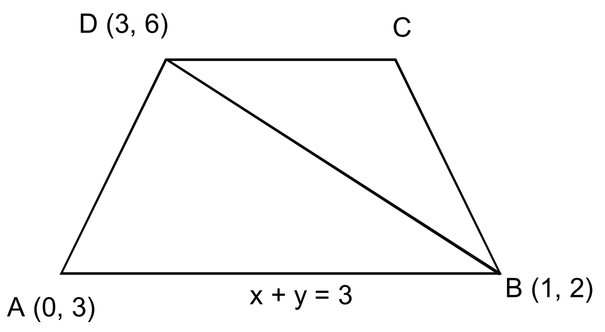
79. Two sides of a parallelogram are along the lines, $x + y = 3$ and $x - y + 3 = 0$. If its diagonals intersect at (2, 4), then one of its vertex is

(1*) (3, 6) (2) (2, 1) (3) (2, 6) (4) (3, 5)

Sol. Intersection point is A (0, 3)

M = (4, 6)

B $\Rightarrow$ (1, 2), D $\rightarrow$ (3, 6)



80. Let A = $\begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where b > 0. Then the minimum value of $\frac{\det.(A)}{b}$ is

(1*) $2\sqrt{3}$ (2) $-2\sqrt{3}$ (3) $-\sqrt{3}$ (4) $\sqrt{3}$

Sol. Det A = $b^2 + 3$

$$\frac{\det A}{b} = b + \frac{3}{b}$$

$\therefore$ Least value $= 2\sqrt{3}$

81. Let f be a differentiable function such that $f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x}$, (x > 0) and f(1) ≠ 4. Then $\lim\limits_{x\to 0^+} xf\left(\frac{1}{x}\right)$

(1) exists and equals $\frac{4}{7}$

(2) exists and equals 0

(3) does not exist

(4*) exists and equals 4

Sol. $f'(x) = 7 - \frac{3}{4}.\frac{f(x)}{x}, x > 0$

$\therefore f'(x) + \frac{3}{4x}f(x) = 7$ (Linear)

$$f(x).e^{\int \frac{3}{4x}dx} = \int 7.e^{\int \frac{3}{4x}dx} + c$$

$f(x).x^{3/4} = \int 7.x^{3/4} + c$

$$= 7\frac{x^{7/4}}{\frac{7}{4}} + c$$

$\therefore f(x) = 4x + cx^{-3/4}$

$$\therefore f\left(\frac{1}{x}\right) = \frac{4}{x} + cx^{3/4}$$

$$\therefore \underset{x\to 0^+}{\text{Lt}}\ xf\left(\frac{1}{x}\right) = \underset{x\to 0^+}{\text{Lt}}\ 4 + cx^{7/4} = 4$$

82. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is

(1) $\sqrt{5}$ (2) $2\sqrt{2}$ (3*) 4 (4) 3

Sol. $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$

Consider constant term

$${}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{\lambda}{x^2}\right)^r$$

$$\frac{10-r}{2}-2r=0$$

$10 - 5r = 0$

$r = 2$

$\Rightarrow {}^{10}C_2 \times \lambda^2 = 720 \Rightarrow \lambda = 4$

83. The plane which bisects the line segment joining the points (– 3, – 3, 4) and (3, 7, 6) at right angles, passes through which one of the following points?

(1) (– 2, 3, 5) (2) (2, 1, 3) (3) (4, – 1, 7) (4*) (4, 1, – 2)

Sol. A (–3, 3, 4), B (3, 7, 6)

Mid point $\Rightarrow$ (0,2,5)

$\vec{n}=\overrightarrow{AB}=6\hat{i}+10\hat{j}+2\hat{k}$

Equation of plane $\vec{r}.\vec{n}=\vec{a}.\vec{n}$

$$\vec{r}.\left(6\hat{i}+10\hat{j}+2\hat{k}\right)=\left(0\hat{i}+2\hat{j}+5\hat{k}\right).\left(6\hat{i}+10\hat{j}+2\hat{k}\right)$$

$3x + 5y + z = 15$

(4, 1, –2) satisfies it

$\therefore$ Answer is D

84. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}\,y + 4\sqrt{2} = 0$ is

(1) $8\sqrt{2}$ (2) $3\sqrt{2}$ (3*) $6\sqrt{3}$ (4) $2\sqrt{11}$

Sol. $x=\sqrt{2}y-4\sqrt{2}$

$x^2 = 4y$

Solving we get point of intersection

$$A\left(-2\sqrt{2},2\right),B\left(4\sqrt{2},8\right)$$

$$\therefore AB=\sqrt{\left(6\sqrt{2}\right)^2+6^2}=6\sqrt{3}$$

85. If $\int x^5 e^{-4x^3}\,dx = \frac{1}{48}e^{-4x^3} = f(x) + C$, where C is a constant of integration, then f(x) is equal to

(1) $4x^3 + 1$ (2*) $-4x^3 - 1$ (3) $-2x^3 - 1$ (4) $-2x^3 + 1$

Sol. $\int x^5 . e^{-4x^3} dx$

$= \int x^2 . x^3 e^{-4x^3} dx$

$-4x^3 = t$

$-12x^2\,dx = dt$

$= \frac{-1}{12}\int -\frac{t}{4}e^t dt$

$= \frac{1}{48}\int t\,e^t dt$

$= \frac{1}{48} te^t - 1.e^t + c$

$= \frac{1}{48} e^{-4x^3}.\left(-4x^3\right) - e^{-4x^3} + c$

86. Let $z = \left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If R(z) and I(z) respectively denote the real and imaginary parts of z, then

(1*) R(z) = – 3 (2) I(z) = 0

(3) R(z) > 0 and I(z) > 0 (4) R(z) < 0 and I(z) > 0

Sol. $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 = \left(e^{i\frac{\pi}{b}}\right)^5 = e^{i5\pi/6}$

$\therefore z = 2\cos\frac{5\pi}{6} = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

87. Consider the following three statements:

P : 5 is a prime number.

Q : 7 is a factor of 192.

R : L.C.M. of 5 and 7 is 35.

Then the truth value of which one of the following statements is true?

(1) $(\sim P) \wedge (\sim Q \wedge R)$ (2*) $P \vee (\sim Q \wedge R)$ (3) $(\sim P) \vee (Q \wedge R)$ (4) $(P \wedge Q) \vee (\sim R)$

Sol. P is True

Q is False

R is True

∴ ~Q is True

~ Q □R is True

∴ $P \vee (\sim Q \wedge R)$ is True.

88. Let $S = \left\{(x,y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$, where $r \neq \pm 1$. Then S represents

(1) an ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, when r > 1.

(2*) an ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when r > 1.

(3) a hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, when 0 < r < 1.

(4) a hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when 0 < r < 1.

Sol. $\frac{y^2}{1+r} - \frac{x^2}{1-r} = 1$

$r > 1 \Rightarrow$ ellipse

$e = \sqrt{1 - \left(\frac{r-1}{r+1}\right)} = \sqrt{\frac{2}{r+1}}$

89. The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is

(1) 1 (2*) 2 (3) $\frac{15}{8}$ (4) $\frac{4}{9}$

Sol. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (3 - \lambda)^2 - 2(2 - \lambda)$

$= \lambda^2 + 9 - 6\lambda - 4 + 2\lambda$

$= \lambda^2 - 4\lambda + 5$

$\therefore$ For least value $\lambda = 2$

90. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations x_1, x_2, x_3, x_4, x_5 and – 50 is equal to

(1) 582.5 (2) 509.5 (3*) 507.5 (4) 586.5

Sol. $\sum x = 50$

$(3)^2 = \frac{1}{5}\left(ex^2 - \frac{(ex)^2}{5}\right)$

$9 = \frac{1}{5}\left(\sum x^2 - \frac{2500}{5}\right)$

$\therefore \sum x^2 = 545$

New variable $= \frac{1}{6}\left(3045 - \frac{0}{6}\right) = 507.5$